

Find $\lim_{x \rightarrow 0} \frac{x^2}{1 - 2x^2 - \cos 2x}$. 0

SCORE: _____ / 4 PTS

Your answer should be a number, ∞ , $-\infty$ or DNE (only if the first three answers do not apply).

$$\begin{aligned} \textcircled{1} &= \boxed{\lim_{x \rightarrow 0} \frac{2x}{-4x + 2\sin 2x}} \quad \frac{0}{0} \\ \textcircled{1} &= \boxed{\lim_{x \rightarrow 0} \frac{2}{-4 + 4\cos 2x}} \quad \frac{2}{0} \\ &= -\infty \\ &\quad \boxed{\textcircled{1} \textcircled{1}} \end{aligned}$$

Find $\lim_{x \rightarrow 0^+} (1 - 4x)^{\cot x}$. 1

SCORE: _____ / 5 PTS

Your answer should be a number, ∞ , $-\infty$ or DNE (only if the first three answers do not apply).

$$\begin{aligned} &\lim_{x \rightarrow 0^+} \ln(1 - 4x)^{\cot x} \\ \textcircled{1} &= \lim_{x \rightarrow 0^+} \cot x \ln(1 - 4x), \quad \infty \cdot 0 \\ \textcircled{1} &= \lim_{x \rightarrow 0^+} \frac{\ln(1 - 4x)}{\tan x}, \quad \frac{0}{0} \\ \textcircled{1} &= \lim_{x \rightarrow 0^+} \frac{\frac{1}{1-4x} \cdot (-4)}{\sec^2 x} \\ &= \frac{-(-4)}{1} = -4 \\ &\quad \boxed{\textcircled{1}} \end{aligned}$$

$$\lim_{x \rightarrow 0^+} (1 - 4x)^{\cot x} = e^{-4}$$

$$\boxed{\textcircled{1}}$$

Graph $f(x) = (1+e^x)^{-2}$ using the process shown in lecture and in the website handout.

SCORE: ____ / 21 PTS

Complete the table at the bottom of the page, after showing relevant work (except for entries marked ★).

You will NOT receive credit for the entries in the table if the relevant work is missing.

$$f(0) = (1+1)^{-2} = 2^{-2} = \frac{1}{4} \quad \text{y-INT } (0, \frac{1}{4}) \quad \text{②}$$

$\frac{1}{(1+e^x)^2}$ IS NEVER 0 NO X-INT ①

$$\lim_{x \rightarrow \infty} \frac{1}{(1+e^x)^2} = 0 \quad (\frac{1}{(1+\infty)^2} \rightarrow 0) \quad \text{①}$$

$$\lim_{x \rightarrow -\infty} \frac{1}{(1+e^x)^2} = \frac{1}{(1+0)^2} = 1 \quad \text{②}$$

$$f'(x) = \frac{-2e^x}{(1+e^x)^3}, \quad \begin{array}{l} \text{IS NEVER UNDEFINED} \\ \text{NOR 0} \end{array} \quad \text{①}$$

$$f''(x) = \frac{-2e^x(1+e^x)^3 + 2e^x(3(1+e^x)^2)e^x}{(1+e^x)^6} \quad \text{②}$$

$$= \frac{2e^x(-1-e^x+3e^x)}{(1+e^x)^4}$$

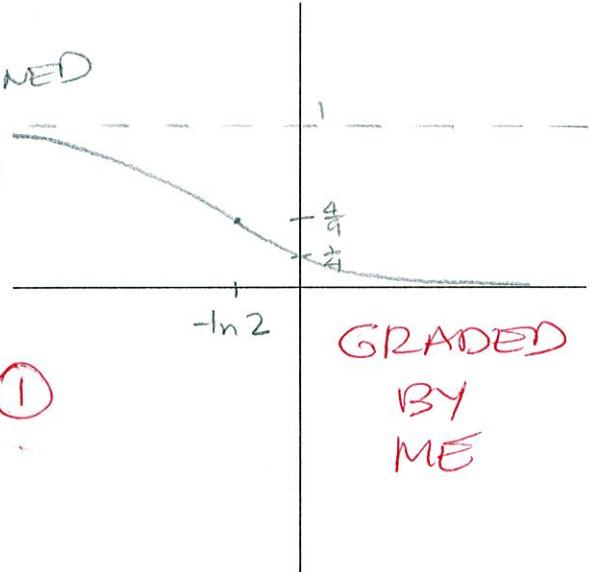
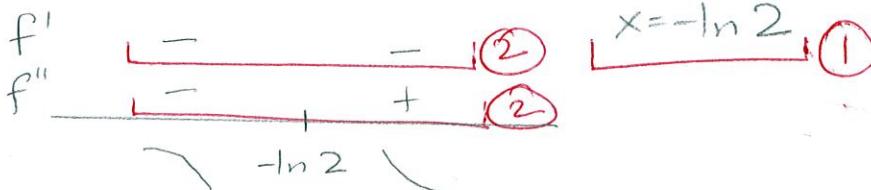
$$= \frac{2e^x(2e^x-1)}{(1+e^x)^4} \quad \begin{array}{l} \text{IS NEVER UNDEFINED} \\ \text{②} \end{array}$$

$$= 0 \text{ IF } 2e^x-1=0$$

$$e^x = \frac{1}{2}$$

$$x = \ln \frac{1}{2}$$

$$x = -\ln 2$$



★ Domain	★ Discontinuities	Intercepts (specify $x-$ or $y-$)	One sided limits at each discontinuity (write using proper limit notation)	
$\text{IR } (1+e^x > 1)$	NONE	$y\text{-INT: } (0, \frac{1}{4})$ $x\text{-INT: NONE}$	N/A	
Horizontal Asymptotes	Intervals of Increase	Intervals of Decrease	Intervals of Upward Concavity	Intervals of Downward Concavity
① $ y=0, 1 $	NONE	$(-\infty, \infty)$	$(0, \infty)$	$(-\infty, 0)$
Vertical Tangent Lines	Horizontal Tangent Lines	Local Maxima	Local Minima	Inflection Points
NONE	NONE	NONE	NONE	$(-\ln 2, \frac{4}{9})$ ①